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Minimal rg*b-Open and Maximal rg*b–Closed Sets

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Abstract: **The objective of this paper is to study the notions of minimal rg*b -closed set, maximal rg*b -open set, minimal rg*b -open set and maximal rg*b -closed set and their basic properties are studied.**

Keywords: **rg*b -closed set and minimal rg*b -closed set, maximal rg*b -open set, minimal rg*b -open set and maximal rg*b -closed set.**

1. INTRODUCTION

Norman Levine [7] introduced the concepts of generalized closed sets in topological spaces. Later in 1996, Andrjivic [2] gave a new type of generalized closed set in topological space called b closed sets. A.A.Omari and M.S.M. Noorani[1] made an analytical study and gave the concepts of generalized b closed sets in topological spaces. The notion of regular generalized star b-closed set and its different characterizations are discussed in [9].

Nakaoka and Oda[4,5,6] have introduced minimal open sets and maximal open sets, which are subclasses of open sets. Later on many authors concentrated in this direction and defined many different types of minimal and maximal open sets. Inspired with these developments we further study a new type of closed and open sets namely minimal rg*b -closed sets, maximal rg*b -open sets, minimal rg*b -open sets and maximal rg*b -closed sets.

Throughout the paper a space X means a topological space (X, τ). The class of rg*b -closed sets is denoted by RG*BC(X). For any subset A of X its complement, interior, closure, rg*b -interior, rg*b -closure are denoted respectively by the symbols A^c, int A, cl(A), Int-rg*b(A), rg*b-cl(A).

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X,τ),is called

1) a **b-open set** [4] if A ⊆ cl (int (A)) ∪ int (cl (A)).
2) a **regular open** set [8] if A = int(cl (A)) and a regular closed set if A = cl(int(A)).
3) a **regular generalized closed set** (briefly, rg-closed)[8] if cl (A) ⊆ U whenever A ⊆ U and U is regular open in X.
4) a **generalized b- closed set** (briefly gb- closed) [2] if bcl (A) ⊆ U whenever A ⊆ U and U is open in X.
5) **a regular generalized b-closed set** (briefly rgb-closed) [3] if bcl(A) ⊆U whenever A ⊆U and U is regular open in X.
6) a **regular Generalized star b- closed set** (briefly rg*b-closed set)[9] if bcl (A) ⊆ U whenever A ⊆U and U is rg-open in X.

Definition 2.2: Let A be a subset of a topological space (X,τ).Then by [10]

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(i) A point x∈A is the rg*b-interior point of A iff ∃ G∈RG*BO(X, τ) such that x∈G⊂A.

(ii) A point is said to be an rg*b-limit point of A iff for each U∈RG*BO(X), U∩(A\{x}) ≠ φ.

(iii) A point x∈A is said to be rg*b-isolated point of A if ∃ U∈ RG*BO(X) such that U∩A = {x}.

Definition 2.3: The set of all rg*b -interior points of A is called the rg*b -interior of A and is denoted by Int- rg*b (A).

Definition 2.4: Let A be a subset of a topological space (X,τ). Then by [10]

(i) A is said to be rg*b-discrete if each point of A is rg*b-isolated point of A.
The set of all rg*b -isolated points of A is denoted by $I_{rg*b}(A)$.

(ii) The intersection of all rg*b-closed sets containing A is called the rg*b-closure of A and is denoted by rg*b-cl(A).

(iii) A \ Int- rg*b (A) is called the rg*b-border or rg*b-boundary of A, and is denoted by b_{rg*b} (A). That is, b_{rg*b} (A) = A \ Int- rg*b (A).

(iv) The rg*b -interior of (X \ A) is called the rg*b -exterior of A, and is denoted by Ext_{rg*b} (A), that is, Ext_{rg*b} (A) = Int- rg*b (X \ A).

Theorem 2.5:

(i) Let $A \subseteq Y \subseteq X$ and Y is regularly open subspace of X then A∈RG*BO(Y, τ/Y) iff Y is rg*b -open in X .

(ii) Let $Y \subseteq X$ and A is a rg*b -neighborhood of x in Y. Then A is rg*b -neighborhood of x in Y iff Y is rg*b -open in X.

Remark 2.6: Finite union and finite intersection of rg*b -closed sets is not rg*b -closed in general.

Theorem 2.7: Let $X = X_1 \times X_2$. Let $A_1 \in RG{*}BC(X_1)$ and $A_2 \in RG{*}BC(X_2)$, then $A_1 \times A_2 \in RG{*}BC(X_1 \times X_2)$.

3. MINIMAL RG*B -OPEN SETS AND MAXIMAL RG*B -CLOSED SETS

We now introduce minimal rg*b -open sets and maximal rg*b -closed sets in topological spaces as follows.

Definition 3.1: A proper nonempty rg*b -open subset U of X is said to be a **Minimal rg*b -open set** if any rg*b -open set contained in U is φ or U.

Example 3.2: Let X = {a, b, c, d}; τ = {φ, {a}, {b,c}, {a, b,c}, X}. Then {a} is both Minimal open and Minimal rg*b -open but {b} and {c} are Minimal rg*b -open but not Minimal open.

Remark 3.3: Minimal open and minimal rg*b -open sets are independent of each other:

Example 3.4: Let X = {a, b, c, d}; τ = {φ, {a, b}, X}. {a, b} is Minimal open but not Minimal rg*b -open and {a}, {b} are Minimal rg*b -open but not Minimal open.

Theorem 3.5:

(i) Let U be a minimal rg*b -open set and W be a rg*b -open set. Then U ∩ W = φ or U⊂W.
(ii)Let U and V be minimal rg*b -open sets. Then U ∩ V = φ or U = V.

Proof:

(i) Let U be a minimal rg*b -open set and W be a rg*b -open set. If U ∩ W = φ, then there is nothing to prove. If U ∩ W ≠ φ. Then U ∩W ⊂U. Since U is a minimal rg*b -open set, we have U ∩ W = U. Therefore U ⊂W.
(ii) Let U and V be minimal rg*b -open sets. If U∩V ≠ φ, then U⊂ V and V⊂ U by (i). Therefore U = V.

Theorem 3.6: Let U be a minimal rg*b -open set. If x∈U, then U⊂W for any regular open neighborhood W of x.

Proof: Let U be a minimal rg*b -open set and x be an element of U. Suppose ∃ a regular open neighborhood W of x such that U ⊄ W. Then U ∩ W is a rg*b -open set such that U ∩ W⊂ U and U ∩ W ≠ φ. Since U is a minimal rg*b -open set, we have U∩ W = U. That is U ⊂W, which is a contradiction for U ⊄ W. Therefore U⊂ W for any regular open neighborhood W of x.

Theorem 3.7: Let U be a minimal rg*b -open set. If x∈U, then U⊂W for some rg*b -open set W containing x.

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Theorem 3.8: Let U be a minimal rg*b -open set. Then U = ∩{W: W∈R*G*BO*(X, x)} for any element x of U.

Proof: By theorem[3.7] and U is rg*b -open set containing x, we have U⊂∩{ W: W∈R*G*BO*(X, x)} ⊂U.

Theorem 3.9: Let U be a nonempty rg*b -open set. Then the following three conditions are equivalent.

(i) U is a minimal rg*b -open set
(ii) U⊂ rg*b-cl(S) for any nonempty subset S of U
(iii) rg*b-cl(U) = rg*b-cl(S) for any nonempty subset S of U.

Proof: (i) ⇒ (ii) Let x∈U; U be minimal rg*b -open set and S(≠ φ) ⊂U. By theorem[3.7], for any rg*b -open set W containing x, S⊂U⊂W ⇒ S⊂W. Now S = S∩U⊂S∩W. Since S≠ φ, S∩W ≠ φ. Since W is any rg*b -open set containing x, x∈ rg*b-cl(S). That is x∈U ⇒ x∈ rg*b-cl(S) ⇒ U⊂ rg*b-cl(S) for any nonempty subset S of U.

(ii) ⇒ (iii) Let S be a nonempty subset of U. That is S⊂U ⇒ rg*b-cl(S) ⊂ rg*b-cl(U) → (1). Again from (ii) U⊂ rg*b-cl(S) for any S(≠ φ) ⊂U ⇒ rg*b-cl(U) ⊂ rg*b-cl(rg*b-cl(S)) = rg*b-cl (S). That is rg*b-cl(U) ⊂ rg*b-cl(S) – → (2). From (1) and (2), we have rg*b-cl(U) = rg*b-cl(S) for any nonempty subset S of U.
(iii) ⇒ (i) From (3) we have rg*b-cl(U) = rg*b-cl(S) for any nonempty subset S of U. Suppose U is not a minimal rg*b -open set. Then ∃ a nonempty rg*b -open set V such that V⊂ U and V ≠ U. Now ∃ an element a in U such that a∉V ⇒ a∈V^c. That is rg*b-cl({a})⊂ rg*b-cl(V^c) = V^c, as V^c is rg*b -closed set in X. It follows that rg*b-cl({a}) ≠ rg*b-cl(U). This is a contradiction for rg*b-cl ({a}) = rg*b-cl(U) for any {a}(≠ϕ)⊂U. Therefore U is minimal rg*b open set.

Theorem 3.10: Let V be a nonempty finite rg*b -open set. Then ∃ atlest one (finite) minimal rg*b open set U such that U⊂V.

Proof: Let V be a nonempty finite rg*b -open set. If V is a minimal rg*b -open set, we may set U = V. If V is not a minimal rg*b -open set, then ∃ (finite) rg*b -open set V_1 such that φ ≠ V_1⊂V. If V_1 is a minimal rg*b -open set, we may set U = V_1. If V_1 is not a minimal rg*b -open set, then ∃ (finite) rg*b -open set V_2 such that φ ≠ V_2 ⊂ V_1. Continuing this process, we have a sequence of rg*b -open sets V ⊃ V_1 ⊃ V_2 ⊃ V_3⊃ ..... ⊃ V_k ⊃ ...... Since V is a finite set, this process repeats only finitely. Then finally we get a minimal rg*b -open set U = V_n for some positive integer n.

[A topological space X is said to be locally finite space if each of its elements is contained in a finite open set.]

Corollary 3.11: Let X be a locally finite space and V be a nonempty rg*b -open set. Then ∃ at least one (finite) minimal rg*b -open set U such that U ⊂ V.

Proof: Let X be a locally finite space and V be a nonempty rg*b -open set. Let x in V. Since X is locally finite space, we have a finite open set V_x such that x in V_x. Then V∩V_x is a finite rg*b -open set. By Theorem 3.10 ∃ at least one (finite) minimal rg*b -open set U such that U ⊂ V∩V_x. That is U⊂V∩V_x⊂V. Hence ∃ at least one (finite) minimal rg*b -open set U such that U⊂V.

Corollary 3.12: Let V be a finite minimal open set. Then ∃ at least one (finite) minimal rg*b -open set U such that U⊂V.

Proof: Let V be a finite minimal open set. Then V is a nonempty finite rg*b -open set. By Theorem 3.10, ∃ at least one (finite) minimal rg*b -open set U such that U⊂V.

Theorem 3.13: Let U; U_λ be minimal rg*b -open sets for any element λ∈Γ. If U⊂$\cup_{\lambda\in\Gamma}U_\lambda$, then ∃ an element λ ∈Γ such that U = U_λ.

Proof: Let U ⊂$\cup_{\lambda\in\Gamma}U_\lambda$. Then U ∩($\cup_{\lambda\in\Gamma}U_\lambda$) = U. That is $\cup_{\lambda\in\Gamma}$(U ∩ U_λ) = U. Also by theorem[3.5] (ii), U ∩ U_λ = φ or U = U_λ for any λ∈Γ. It follows that ∃ an element λ∈Γ such that U = U_λ.

Theorem 3.14: Let U; U_λ be minimal rg*b -open sets for any λ∈Γ. If U = U_λ for any λ∈Γ, then ($\cup_{\lambda\in\Gamma}U_\lambda$) ∩ U = φ.

Proof: Suppose that ($\cup_{\lambda\in\Gamma}U_\lambda$) ∩ U≠ φ. That is $\cup_{\lambda\in\Gamma}$(U_λ ∩ U) ≠ φ. Then ∃ an element λ∈Γ such that U ∩ U_λ ≠ φ. By theorem 3.5(ii), we have U = U_λ, which contradicts the fact that U ≠ U_λ for any λ∈Γ. Hence ($\cup_{\lambda\in\Gamma}U_\lambda$)∩U = φ.

We now introduce **Maximal rg*b -closed sets** in topological spaces as follows.

Definition 3.15: A proper nonempty rg*b -closed F⊂X is said to be **maximal** rg*b **-closed set** if any rg*b -closed set containing F is either X or F.

Example 3.16: In Example 3.2, {b, c, d} is both Maximal closed and Maximal rg*b -closed but {a, b, c} and {a, b, d} are Maximal rg*b -closed but not Maximal closed.

Remark 3.17: Maximal closed and maximal rg*b -closed sets are independent of each other:

Example 3.18: In Example 3.4, {c} is Maximal closed but not Maximal rg*b -closed and {a, c} and {b,c} are Maximal rg*b -closed but not Maximal closed.

Remark 3.19: From the known results and by the above example we have the following implications:

Theorem 3.20: A proper nonempty subset F of X is maximal rg*b -closed set iff X-F is a minimal rg*b -open set.

Proof*:* Let F be a maximal rg*b -closed set. Suppose X-F is not a minimal rg*b -open set. Then $\exists$ rg*b -open set $U \neq X-F$ such that $\varphi \neq U \subset X-F$. That is $F \subset X-U$ and X-U is a rg*b -closed set which is a contradiction for F is a maximal rg*b -closed set.

Conversely let X-F be a minimal rg*b -open set. . Suppose F is not a maximal rg*b –closed set, then $\exists$ rg*b closed set $E \neq F$ such that $F \subset E \neq X$. That is $\varphi \neq X-E \subset X-F$ and X-E is a rg*b -open set which is a contradiction for X-F is a minimal rg*b -open set. Therefore F is a maximal rg*b -closed set.

Theorem 3.21:

(i) Let F be a maximal rg*b -closed set and W be a rg*b -closed set. Then $F \cup W = X$ or $W \subset F$.
(ii) Let F and S be maximal rg*b -closed sets. Then $F \cup S = X$ or $F = S$.

Proof*:* (i) Let F be a maximal rg*b -closed set and W be a rg*b -closed set. If $F \cup W = X$, then there is nothing to prove. Suppose $F \cup W \neq X$. Then $F \subset F \cup W$. Therefore $F \cup W = F \Rightarrow W \subset F$.

(ii) Let F and S be maximal rg*b -closed sets. If $F \cup S \neq X$, then we have $F \subset S$ and $S \subset F$ by (i). Therefore $F = S$.

Theorem 3.22: Let F be a maximal rg*b -closed set. If x is an element of F, then for any rg*b -closed set S containing x, $F \cup S = X$ or $S \subset F$.

Proof*:* Let F be a maximal rg*b -closed set and x is an element of F. Suppose $\exists$ rg*b -closed set S containing x such that $F \cup S \neq X$. Then $F \subset F \cup S$ and let $F \cup S$ is a rg*b -closed set. Since F is a rg*b -closed set, we have $F \cup S = F$. Therefore $S \subset F$.

Theorem 3.23*:* Let F_α, F_β, F_δ be maximal rg*b -closed sets such that $F_\alpha \neq F_\beta$. If $F_\alpha \cap F_\beta \subset F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$

Proof*:* Given that $F_\alpha \cap F_\beta \subset F_\delta$. If $F_\alpha = F_\delta$ then there is nothing to prove.

If $F_\alpha \neq F_\delta$ then we have to prove $F_\beta = F_\delta$. Now $F_\beta \cap F_\delta = F_\beta \cap (F_\delta \cap X) = F_\beta \cap (F_\delta \cap (F_\alpha \cup F_\beta)$(by thm. 3.21 (ii)) $= F_\beta \cap ((F_\delta \cap F_\alpha) \cup (F_\delta \cap F_\beta)) = (F_\beta \cap F_\delta \cap F_\alpha) \cup (F_\beta \cap F_\delta \cap F_\beta)$
$= (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta)$ (by $F_\alpha \cap F_\beta \subset F_\delta$) $= (F_\alpha \cup F_\delta) \cap F_\beta = X \cap F_\beta$ (Since F_α and F_δ are maximal rg*b -closed sets by theorem[3.21](ii), $F_\alpha \cup F_\delta = X$) $= F_\beta$. That is $F_\beta \cap F_\delta = F_\beta \Rightarrow F_\beta \subset F_\delta$ Since F_β and F_δ are maximal rg*b -closed sets, we have $F_\beta = F_\delta$ Therefore $F_\beta = F_\delta$

Theorem 3.24: Let F_α, F_β and F_δ be different maximal rg*b -closed sets to each other. Then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Proof*:* Let $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\delta) \Rightarrow (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta) \subset (F_\alpha \cap F_\delta) \cup (F_\delta \cap F_\beta) \Rightarrow (F_\alpha \cup F_\delta) \cap F_\beta \subset F_\delta \cap (F_\alpha \cup F_\beta)$. Since by theorem 3.21(ii), $F_\alpha \cup F_\delta = X$ and $F_\alpha \cup F_\beta = X \Rightarrow X \cap F_\beta \subset F_\delta \cap X \Rightarrow F_\beta \subset F_\delta$ From the definition of maximal rg*b -closed set it follows that $F_\beta = F_\delta$, which is a contradiction to the fact that F_α, F_β and F_δ are different to each other. Therefore $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Theorem 3.25: Let F be a maximal rg*b -closed set and x be an element of F. Then $F = \cup$ { S: S is a rg*b -closed set containing x such that $F \cup S \neq X$}.

Proof: By theorem 3.23 and fact that F is a rg*b -closed set containing x, we have $F \subset \cup$ { S: S is a rg*b -closed set containing x such that $F \cup S \neq X$} – F. Therefore we have the result.

Theorem 3.26: Let F be a proper nonempty cofinite rg*b -closed set. Then $\exists$ (cofinite) maximal rg*b -closed set E such that $F \subset E$.

Proof: If F is maximal rg*b -closed set, we may set E = F. If F is not a maximal rg*b -closed set, then ∃ (cofinite) rg*b -closed set F_1 such that $F \subset F_1 \neq X$. If F_1 is a maximal rg*b -closed set, we may set $E = F_1$. If F_1 is not a maximal rg*b -closed set, then ∃ a (cofinite) rg*b -closed set F_2 such that $F \subset F_1 \subset F_2 \neq X$. Continuing this process, we have a sequence of rg*b closed, $F \subset F_1 \subset F_2 \subset - - - \subset F_k - - - - -$ .Since F is a cofinite set ,this process repeats only finitely. Then, finally we get a maximal rg*b -closed set $E = E_n$ for some positive integer n.

Theorem 3.27: Let F be a maximal rg*b -closed set. If x is an element of X-F. Then X-F⊂ E for any rg*b -closed set E containing x.

Proof: Let F be a maximal rg*b -closed set and x in X-F. E ⊄ F for any rg*b -closed set E containing x. Then E ∪ F = X by theorem 3.21(ii). Therefore X-F ⊂E.

4. MINIMAL RG*B -CLOSED SET AND MAXIMAL RG*B -OPEN SET

We now introduce Minimal rg*b -closed sets and Maximal rg*b -open sets in topological spaces as follows.

Definition 4.1: A proper nonempty rg*b -closed subset F of X is said to be a **Minimal** rg*b **-closed set** if any rg*b -closed set contained in F is φ or F.

Example 4.2: In Example 3.2**,** {d} is both a Minimal closed and Minimal rg*b -closed set.

Remark 4.3: Minimal closed and minimal rg*b -closed sets are independent to each other:

Example 4.4: In Example 3.4, {c} is Minimal closed but not Minimal rg*b -closed set and {a} and {b} are Minimal rg*b -closed but not Minimal closed.

Definition 4.5: A proper nonempty rg*b -open U⊂X is said to be a **Maximal** rg*b **-open set** if any rg*b -open set containing U is either X or U.

Example 4.6: In Example 3.4, {a,b} is both maximal open and maximal rg*b open.

Remark 4.7: Maximal open set and maximal rg*b -open set are independent to each other.

Example 4. 8: In Example 3.2. {a, b,c} is Maximal open but not maximal rg*b -open and {a, b, d} and {a, c, d} are Maximal rg*b -open but not maximal open.

Theorem 4.9: A proper nonempty subset U of X is maximal rg*b -open set iff X-U is a minimal rg*b -closed set.

Proof: Let U be a maximal rg*b -open set. Suppose X-U is not a minimal rg*b -closed set. Then ∃ rg*b -closed set V ≠ X-U such that φ ≠ V ⊂X-U. That is U⊂X-V and X-V is a rg*b -open set which is a contradiction for U is a minimal rg*b -closed set. Conversely let X-U be a minimal rg*b -closed set. Suppose U is not a maximal rg*b -open set. Then ∃ a rg*b -open set E ≠ U such that U⊂ E ≠ X. That is φ ≠ X-E ⊂X-U and X-E is a rg*b -closed set which is a contradiction for X-U is a minimal rg*b -closed set. Therefore U is a maximal rg*b -closed set.

Lemma 4.10:

(i) Let U be a minimal rg*b -closed set and W be a rg*b - closed set. Then U ∩ W = φ or U ⊂ W.
(ii) Let U and V be minimal rg*b - closed sets. Then U ∩ V = φ or U = V.

Proof:

(i) Let U be a minimal rg*b -closed set and W be a rg*b -closed set. If U ∩ W = φ, then there is nothing to prove.
If U ∩ W ≠ φ. Then U ∩W ⊂U. Since U is a minimal rg*b -closed set, we have U ∩ W = U. Therefore U ⊂W.
(ii) Let U and V be minimal rg*b -closed sets. If U∩V ≠ φ, then U⊂V and V⊂ U by (i). Therefore U = V.

Theorem 4.11: Let U be a minimal rg*b -closed set. If x∈ U, then U⊂W for any regular open neighborhood W of x.

Proof: Let U be a minimal rg*b -closed set and x be an element of U. Suppose ∃ an regular open neighborhood W of x such that U ⊄ W. Then U ∩ W is a rg*b -closed set such that U ∩ W⊂ U and U ∩ W ≠ φ. Since U is a minimal rg*b -closed set, we have U∩ W = U. That is U ⊂W, which is a contradiction for U ⊄ W. Therefore U⊂W for any regular open neighborhood W of x.

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Theorem 4.12: Let U be a minimal rg*b -closed set. If $x\in U$, then $U\subset W$ for some rg*b-closed set W containing x.

Theorem 4.13: Let U be a minimal rg*b -closed set. Then $U = \cap\{ W: W\in RG*BC(X, x)\}$ for any element x of U

Proof: By theorem[4.12] and U is rg*b -closed set containing x, we have $U\subset\cap\{ W: W\in RG*BC(X, x)\} \subset U$.

Theorem 4.14: Let U be a nonempty rg*b -closed set. Then the following three conditions are equivalent.

(i) U is a minimal rg*b -closed set
(ii) $U\subset$ rg*b-cl(S) for any nonempty subset S of U
(iii) rg*b-cl(U) = rg*b-cl(S) for any nonempty subset S of U.

Proof: (i) $\Rightarrow$ (ii) Let $x\in U$; U be minimal rg*b -closed set and $S(\neq \varphi) \subset U$. By theorem[4.12], for any rg*b -closed set W containing x, $S\subset U\subset W \Rightarrow S\subset W$. Now $S = S\cap U\subset S\cap W$. Since $S\neq \varphi$, $S\cap W \neq \varphi$. Since W is any rg*b -closed set containing x, by theorem[4.12], $x\in$ rg*b-cl(S). That is $x\in U \Rightarrow x\in$ rg*b-cl(S) $\Rightarrow U \subset$ rg*b-cl(S) for any nonempty subset S of U.

(ii) $\Rightarrow$ (iii) Let S be a nonempty subset of U. That is $S\subset U \Rightarrow$ rg*b-cl(S) $\subset$ rg*b-cl(U) $\rightarrow$ (1). Again from (ii) $U\subset$ rg*b-cl(S) for any $S(\neq \varphi) \subset U \Rightarrow$ rg*b-cl(U) $\subset$ rg*b-cl(rg*b-cl(S)) = rg*b-cl (S). That is rg*b-cl(U) $\subset$ rg*b-cl(S) $\rightarrow$ (2). From (1) and (2), we have rg*b-cl(U) = rg*b-cl(S) for any nonempty subset S of U.
(iii) $\Rightarrow$ (i) From (3) we have rg*b-cl(U) = rg*b-cl(S) for any nonempty subset S of U. Suppose U is not a minimal rg*b -closed set. Then $\exists$ a nonempty rg*b -closed set V such that $V \subset U$ and $V \neq U$. Now $\exists$ an element a in U such that $a\notin V \Rightarrow a\in V^c$. That is rg*b-cl({a}) $\subset$ rg*b-cl(V^c) = V^c, as V^c is rg*b -closed set in X. It follows that rg*b-cl({a}) $\neq$ rg*b-cl(U). This is a contradiction for rg*b-cl({a}) = rg*b-cl(U) for any $\{a\}(\neq \varphi) \subset U$. Therefore U is a minimal rg*b -closed set.

Theorem 4.15: Let V be a nonempty finite rg*b -closed set. Then $\exists$ at least one (finite) minimal rg*b -closed set U such that $U\subset V$.

Proof: Let V be a nonempty finite rg*b-closed set. If V is a minimal rg*b-closed set, we may set U = V. If V is not a minimal rg*b -closed set, then $\exists$ (finite) rg*b -closed set V_1 such that $\varphi \neq V_1\subset V$. If V_1 is a minimal rg*b -closed set, we may set $U = V_1$. If V_1 is not a minimal rg*b -closed set, then $\exists$ (finite) rg*b -closed set V_2 such that $\varphi \neq V_2 \subset V_1$. Continuing this process, we have a sequence of rg*b -closed sets $V \supset V1 \supset V2 \supset V3 \supset \ldots\ldots \supset Vk \supset \ldots\ldots$ Since V is a finite set, this process repeats only finitely. Then finally we get a minimal rg*b -closed set $U = V_n$ for some positive integer n.

Corollary 4.16: Let X be a locally finite space and V be a nonempty rg*b -closed set. Then $\exists$ at least one (finite) minimal rg*b -closed set U such that $U \subset V$.

Proof: Let X be a locally finite space and V be a nonempty rg*b -closed set. Let x in V. Since X is locally finite space, we have a finite open set V_x such that x in V_x. Then $V\cap V_x$ is a finite rg*b -closed set. By Theorem 4.15 $\exists$ at least one (finite) minimal rg*b -closed set U such that $U\subset V\cap V_x$. That is $U\subset V\cap V_x\subset V$. Hence $\exists$ at least one (finite) minimal rg*b -closed set U such that $U\subset V$.

Corollary 4.17: Let V be a finite minimal open set. Then $\exists$ at least one (finite) minimal rg*b -closed set U such that $U\subset V$.

Proof: Let V be a finite minimal open set. Then V is a nonempty finite rg*b -closed set. By Theorem 4.15, $\exists$ at least one (finite) minimal rg*b -closed set U such that $U\subset V$.

Theorem 4.18: Let U; U_λ be minimal rg*b -closed sets for any element $\lambda\in\Gamma$. If $U\subset\cup_{\lambda\in\Gamma}U_\lambda$, then $\exists$ an element $\lambda \in\Gamma$ such that $U = U_\lambda$.

Proof: Let $U \subset\cup_{\lambda\in\Gamma}U_\lambda$. Then $U \cap(\cup_{\lambda\in\Gamma}U_\lambda) = U$. That is $\cup_{\lambda\in\Gamma}(U \cap U_\lambda) = U$. Also by lemma[4.10] (ii), $U \cap U_\lambda = \varphi$ or $U = U_\lambda$ for any $\lambda\in\Gamma$. It follows that $\exists$ an element $\lambda\in\Gamma$ such that $U = U_\lambda$

Theorem 4.19: Let U; U_λ be minimal rg*b -closed sets for any $\lambda\in\Gamma$. If $U = U_\lambda$ for any $\lambda\in\Gamma$, then $(\cup_{\lambda\in\Gamma}U_\lambda) \cap U = \varphi$.

Proof: Suppose that $(\cup_{\lambda\in\Gamma}U_\lambda) \cap U\neq \varphi$. That is $\cup_{\lambda\in\Gamma}(U_\lambda \cap U) \neq \varphi$. Then $\exists$ an element $\lambda\in\Gamma$ such that $U \cap U_\lambda \neq \varphi$. By lemma[4.1](ii), we have $U = U_\lambda$, which contradicts the fact that $U \neq U_\lambda$ for any $\lambda\in\Gamma$. Hence $(\cup_{\lambda\in\Gamma}U_\lambda)\cap U = \varphi$.

Theorem 4.20: A proper nonempty subset F of X is maximal rg*b -open set iff X-F is a minimal rg*b -closed set.

***Proof**:* Let F be a maximal rg*b -open set. Suppose X-F is not a minimal rg*b -closed set. Then ∃ rg*b -closed set U ≠ X-F such that φ ≠ U ⊂X-F. That is F⊂ X-U and X-U is a rg*b -open set which is a contradiction for F is a maximal rg*b -open set.

Conversely let X-F be a minimal rg*b -closed set. Suppose F is not a maximal rg*b -open set. Then ∃ rg*b -open set E ≠ F such that F ⊂ E ≠ X. That is φ ≠ X-E⊂ X-F and X-E is a rg*b -closed set which is a contradiction for X-F is a minimal rg*b -closed set. Therefore F is a maximal rg*b -open set.

Theorem 4.21:

(i) Let F be a maximal rg*b -open set and W be a rg*b -open set. Then F∪W = X or W⊂F.
(ii) Let F and S be maximal rg*b -open sets. Then F ∪ S = X or F = S.

***Proof**:* (i) Let F be a maximal rg*b -open set and W be a rg*b -open set. If F∪ W = X, then there is nothing to prove. Suppose F ∪ W ≠ X. Then F ⊂F ∪ W. Therefore F∪W = F ⇒ W⊂F.

(ii) Let F and S be maximal rg*b -open sets. If F∪S ≠ X, then we have F⊂S and S⊂F by (i). Therefore F = S.

Theorem 4.22: Let F be a maximal rg*b -open set. If x is an element of F, then for any rg*b -open set S containing x, F ∪ S = X or S ⊂F.

***Proof**:* Let F be a maximal rg*b -open set and x is an element of F. Suppose ∃ rg*b -open set S containing x such that F ∪ S ≠ X. Then F⊂ F ∪ S and let F ∪ S is a rg*b -open set. Since F is a rg*b -open set, we have F ∪ S = F. Therefore S⊂ F.

***Theorem 4.23**:* Let F_α, F_β, F_δ be maximal rg*b -open sets such that $F_\alpha \neq F_\beta$. If $F_\alpha \cap F_\beta \subset F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$

***Proof**:* Given that $F_\alpha \cap F_\beta \subset F_\delta$. If $F_\alpha = F_\delta$ then there is nothing to prove.

If $F_\alpha \neq F_\delta$ then we have to prove $F_\beta = F_\delta$. Now $F_\beta \cap F_\delta = F_\beta \cap (F_\delta \cap X) = F_\beta \cap (F_\delta \cap (F_\alpha \cup F_\beta)$(by thm. 4.21 (ii)) $= F_\beta \cap ((F_\delta \cap F_\alpha) \cup (F_\delta \cap F_\beta)) = (F_\beta \cap F_\delta \cap F_\alpha) \cup (F_\beta \cap F_\delta \cap F_\beta)$
$= (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta)$ (by $F_\alpha \cap F_\beta \subset F_\delta$) $= (F_\alpha \cup F_\delta) \cap F_\beta = X \cap F_\beta$ (Since F_α and F_δ are maximal rg*b -open sets by theorem[4.21](ii), $F_\alpha \cup F_\delta = X$) $= F_\beta$. That is $F_\beta \cap F_\delta = F_\beta \Rightarrow F_\beta \subset F_\delta$ Since F_β and F_δ are maximal rg*b -open sets, we have $F_\beta = F_\delta$ Therefore $F_\beta = F_\delta$

Theorem 4.24: Let F_α, F_β and F_δ be different maximal rg*b -open sets to each other. Then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

***Proof**:* Let $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\delta) \Rightarrow (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta) \subset (F_\alpha \cap F_\delta) \cup (F_\delta \cap F_\beta) \Rightarrow (F_\alpha \cup F_\delta) \cap F_\beta \subset F_\delta \cap (F_\alpha \cup F_\beta)$. Since by theorem 4.21(ii), $F_\alpha \cup F_\delta = X$ and $F_\alpha \cup F_\beta = X \Rightarrow X \cap F_\beta \subset F_\delta \cap X \Rightarrow F_\beta \subset F_\delta$ From the definition of maximal rg*b -open set it follows that $F_\beta = F_\delta$, which is a contradiction to the fact that F_α, F_β and F_δ are different to each other. Therefore $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Theorem 4.25: Let F be a maximal rg*b -open set and x be an element of F. Then F = ∪ { S: S is a rg*b -open set containing x such that F ∪ S ≠ X}.

***Proof**:* By theorem 4.23 and fact that F is a rg*b -open set containing x, we have F⊂{ S: S is a rg*b -open set containing x such that F ∪ S ≠ X} ⊂F. Therefore we have the result.

***Theorem 4.26**:* Let F be a proper nonempty cofinite rg*b -open set. Then ∃ (cofinite) maximal rg*b -open set E such that F ⊂E.

***Proof**:* If F is maximal rg*b -open set, we may set E = F. If F is not a maximal rg*b -open set, then ∃ (cofinite) r*g*b*-open set F_1 such that $F \subset F_1 \neq X$. If F_1 is a maximal rg*b -open set, we may set $E = F_1$. If F_1 is not a maximal rg*b -open set, then ∃ a (cofinite) rg*b -open set F_2 such that $F \subset F_1 \subset F_2 \neq X$. Continuing this process, we have a sequence of rg*b -open, $F \subset F_1 \subset F_2 \subset \ldots F_k \subset \ldots \ldots$ Since F is a cofinite set, this process repeats only finitely. Then, finally we get a maximal rg*b -open set $E = E_n$ for some positive integer n.

***Theorem 4.27**:* Let F be a maximal rg*b -open set. If x is an element of X-F. Then X-F ⊂E for any rg*b -open set E containing x.

Proof: Let F be a maximal rg*b -open set and x in X-F. E $\not\subset$ F for any rg*b open set E containing x.Then E$\cup$F=X by theorem 4.21(ii).Therefore X-F$\subset$ E.

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